Approximate Algorithms for Maximizing the Capacity of the Reverse Link in Multiple–Class CDMA Systems

Arash Abadpour and Attahiru Sule Alfa
Department of Electrical and Computer Engineering, University of Manitoba

11th INFORMS Computing Society Conference

Outline

Introduction: CDMA

- Code Division Multiple Access (CDMA): Efficient and stable means of communication between a group of users which share the same physical medium.
- Several independent users access a common communication medium by modulating their symbols with preassigned spreading sequences.
- Multiple Access Interference (MIA) has to be,
  - Suppressed through the implementation of advanced signal processing methods and receiver beam–forming.
  - Managed through efficient power control.
- Here, we work on the *management of capacity* in a single–cell system *through power control*.

Introduction: Aggregate Capacity Maximization

- **Basic Approach**: To define a set of constraints and then to find the solution which is binding for all of them.
  - Constant Signal to Interference Ratio (SIR).
  - Through open–loop power control by individual stations as guided by power messages transmitted by the base station.
- **Multimedia–Enabled Networks**: Maximization of the aggregate capacity becomes the goal.
  - A set of constraints have to be satisfied.
- The challenge is to develop an appropriate formulation and then subsequently to solve it.
- Here, the reverse link (uplink), more precisely speaking, the traffic channels on the reverse link, are considered.

Introduction: Constraints

- Minimum and maximum transmission power model the physical limitations of the mobile stations.
- Maximum received power at the base stations ensures minimal interference with the surrounding cells.
- Minimum SIR is a necessary constraint, due to technical reasons.
  - In the absence of more elaborate constraints, minimum SIR has shown to result in largely unfair systems.
- Maximum bound on the capacities of the individual stations is shown to put a limit on the unfairness of the system.

Unfairness of the System

Difference/Ratio between the highest and the lowest capacities offered in the system.

Introduction: Classes of Service

- Majority of the previous works use the same bounds for all the stations.
- Service–providers tend to offer different levels of service at different premium rates.
- Mobile stations may have different characteristics, for example operating at different battery levels.
- The customization of the problem in the way that all the bounds are personal to the stations produces a problem the available methods are not able to solve.
  - In this work, we formulate the capacity maximization problem within a multiple–class framework.
  - Approximations are utilized in order to estimate the problem as first– and second–order optimization tasks.
  - A solver is proposed and analyzed.

System Model

- There are M mobile stations with reverse link gains of $g_1, \cdots, g_M$.
- p_i: The transmit power of the i–th mobile station.
- p_i is bounded by p_i^{max},

$$0 \leq p_i \leq p_i^{max}, \forall i.$$

- I: Background noise.
- γ_i: SIR for the signal coming from the i–th mobile station, as perceived by the base station,

$$\gamma_i = \frac{p_i g_i}{I + \sum_{j=1, j \neq i}^{M} p_j g_j}.$$

System Model, continued

- We assume that Shannon's formula can be used to approximately relate SIR to the bandwidth (Details in the paper),

$$C_i = B \log_2 (1 + \gamma_i)$$

  - C_i: Capacity of the i–th mobile station.
- The constant B is omitted for notational convenience
  - Relative capacities are analyzed here.

System Model: Problem Formulation

- The problem is defined as maximizing,

$$C = \sum_{i=1}^{M} \alpha_i C_i,$$

- subject to,

$$\begin{cases} \gamma_i \geq \gamma_i^{min}, \forall i, \\ C_i \leq C_i^{max}, \forall i, \\ \sum_{i=1}^{M} p_i g_i \leq P^{max}, \\ 0 \leq p_i \leq p_i^{max}, \forall i. \end{cases}$$

Naming Convention

We call this problem the Multiple–Class Single–Cell (MSC).

System Model: More Parameters

- γ_i^{min}: Minimum SIR for the the i–th mobile station.
- C_i^{max}: Maximum capacity for the the i–th mobile station.
- α_i: Significance of the i–th mobile station i ($\alpha_i > 0$).
- Through grouping the stations into classes of identical values for these parameters, this model will be applicable to a multiple–class scenario.
- Setting $\alpha_i = 1$, $\gamma_i^{min} = \gamma$, $C_i^{max} = \eta$, and $p_i^{max} = p_{max}$, this problem will reduce to the single–class problem known as the NSC.
- Solution to the NSC can be precisely calculated in $O(M^3)$ flops.
- New methodology is needed for solving the MSC.

Proposed Method: Outline

- A set of substitute variables will be proposed.
- The constraints are written as linear forms in terms of the substitute variables.
- The objective function is approximated as linear and quadratic functions of the substitute variables.
- The resulting problem is solved.
- Back transformation approximately produces the optimal transmission powers.

Proposed Method: Substitute Variables

- We define,

$$\varphi_i = \frac{\gamma_i}{1+\gamma_i} = \frac{p_i g_i}{\sum_{j=1}^{M} p_j g_j + I}, \forall i.$$

- Now,

$$C_i = -\log_2\left(1-\varphi_i\right).$$

- And,

$$p_i g_i = I \frac{\varphi_i}{1-\sum_{j=1}^{M}\varphi_j}.$$

- The constraints are rewritten as,

$$\begin{cases} \varphi_i^{min} \leq \varphi_i \leq \varphi_i^{max}, \forall i, \\ \sum\limits_{i=1}^{M} \varphi_i \leq \frac{X^{max}}{X^{max}+1}, \\ l_i \sum\limits_{j=1}^{M} \varphi_j + \varphi_i \leq l_i, \forall i. \end{cases}$$

Proposed Method: Substitute Variables, continued

- Here,

$$\begin{cases} \varphi_i^{min} = \dfrac{\gamma_i^{min}}{\gamma_i^{min}+1}, \\ \varphi_i^{max} = 1 - 2^{-C_i^{max}}, \\ X^{max} = \dfrac{P^{max}}{I}, \\ l_i = \dfrac{p_i^{max} g_i}{I}. \end{cases}$$

Proposed Method: Canonical Representation of the Constraints

- The constraints can be written as,

$$\begin{cases} \vec{\varphi^{\mathbf{min}}} \leq \vec{\varphi} \leq \vec{\varphi^{\mathbf{max}}}, \\ \mathbf{A}\vec{\varphi} \leq \vec{\mathbf{b}}. \end{cases}$$

- Where,

$$\mathbf{A} = \begin{bmatrix} \vec{\mathbf{1}}_{1\times M} \\ \mathbf{1}_{M\times M} + diag\left[\frac{1}{l_1}, \cdots, \frac{1}{l_M}\right] \end{bmatrix},$$

$$\vec{\mathbf{b}} = \begin{bmatrix} \dfrac{X^{max}}{X^{max}+1} \\ \vec{\mathbf{1}}_{M\times 1} \end{bmatrix}.$$

Proposed Method: Approximating the Objective Function

- For small γ_i,

$$C_i = \log_2(1+\gamma_i) \simeq \frac{1}{ln2}\gamma_i \simeq \frac{1}{ln2}\varphi_i.$$

  - Here, we have used,

$$ln(1+x) \simeq \frac{x}{1+x}, x \in [\gamma, 2^\eta - 1],$$

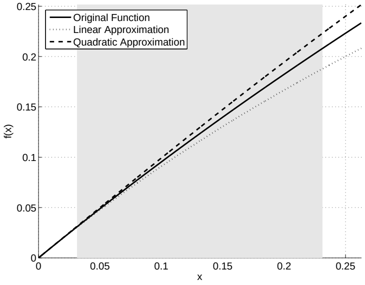
- Again, for small γ_i,

$$C_i \simeq \frac{1}{ln2}\gamma_i = \frac{1}{ln2}\frac{\varphi_i}{1-\varphi_i} \simeq \frac{1}{ln2}\varphi_i(1+\varphi_i).$$

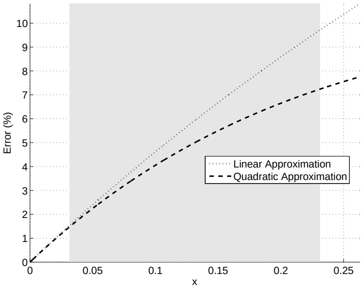
  - Here, we have used,

$$ln(1+x) \simeq \frac{x}{1+x}\left(1+\frac{x}{1+x}\right), x \in [\gamma, 2^\eta - 1].$$

Proposed Method: Approximating the Objective Function, Appropriateness Test



Original Function and the Approximations



Relative Error

- Nominal values of $\gamma = -30dB$ and $\eta = 0.3$ (the shaded area).
- Both approximations induce less than 10% error.
- As p_i increases, and thus so do γ_i and φ_i, the error goes up.
- The linear approximation is *conservative*, the quadratic form *underestimates*.

Proposed Method: Canonical Representation of the Objective Function

- Using the linear approximation,

$$C \simeq \frac{1}{ln2}\sum_{i=1}^{M}\alpha_i\varphi_i = \vec{\mathbf{f}}^T\vec{\varphi}.$$

- Here,

$$\vec{\mathbf{f}} = \frac{1}{ln2}\vec{\alpha}.$$

- Using the quadratic approximation,

$$C \simeq \frac{1}{ln2}\sum_{i=1}^{M}\alpha_i\left(\varphi_i + \varphi_i^2\right) = \frac{1}{2}\vec{\varphi}^T\mathbf{H}\vec{\varphi} + \vec{\mathbf{f}}^T\vec{\varphi},$$

- Here,

$$\mathbf{H} = \frac{2}{ln2}diag\left[\alpha_1, \cdots, \alpha_M\right].$$

Proposed Method: Addition of Other Constraints

- In the previous approach, the addition of new constraints demanded a complete overhaul of the method.
- Here, the steps for embedding the maximum capacity share constraint,

$$\tilde{C}_i = \frac{C_i}{C} \leq \frac{1}{\mu}\frac{1}{M}, 0 < \mu < 1,$$

  are presented.
- The constraint is approximated as,

$$\sum_{j=1}^{M} \alpha_j \vec{\varphi}_j \geq M\mu\varphi_i, \forall i.$$

- Thus leading to the linear form.

$$\left(M\mu\vec{\mathbf{I}}_{M\times M} - \vec{\mathbf{1}}_{M\times 1}\vec{\alpha}^T \right) \vec{\varphi} \leq \vec{\mathbf{0}}_{M\times 1}.^{1}$$

[1] Correction on the paper.

Sketch of the Proposed Algorithm

- Generate $\mathbf{A}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{f}}$ and $\mathbf{H}$,
- Solve either,

  $$\vec{\varphi}=\text{linprog}(\vec{\mathbf{f}},\mathbf{A},\vec{\mathbf{b}},\vec{\varphi}^{min},\vec{\varphi}^{max}),$$

  for the case of the M^1SC, or

  $$\vec{\varphi}=\text{quadprog}(\mathbf{H},\vec{\mathbf{f}},\mathbf{A},\vec{\mathbf{b}},\vec{\varphi}^{min},\vec{\varphi}^{max}),$$

  for the case of the M^2SC.
- Recalculate C_is and C using the exact formulation.
- Calculate p_is.

Naming Convention and Computational Complexity

- M^nSC: Solving the MSC using n–th degree approximation of the objective function.
- As $\mathbf{H}$ is positive–definite, the computational complexity of the M^2SC is polynomial.
- M^1SC, will take up polynomial time as well.

Experimental Results: System Specifications

- Implemented in MATLAB 7.0 and executed on a Pentium IV 3.00GHZ personal computer with 1GB of RAM running Windows xp.
- In a circular cell of radius $R = 2.5Km$.
- d_i: The distance from the i–th mobile station to the base station.
- We use the path loss model,

$$g_i = Cd_i^n,$$

$$C = 7.75 \times 10^{-3}, n = -3.66.$$

- System Parameters,

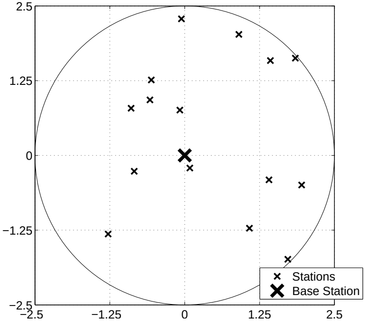
$$\gamma = -30dB, I = -113dBm$$

$$P_{max} = -113dBm, p_{max} = 23dBm$$

$$\eta = 0.3$$

Experiment I: MSC vs. NSC

- A 15–station cell is used.
- In order to be able to apply the NSC and the proposed algorithms on the same problem, we set $\alpha_i = 1$, $\gamma_i^{min} = \gamma$, $C_i^{max} = \eta$, and $p_i^{max} = p_{max}$, for all the mobile stations.



Experiment I: MSC vs. NSC

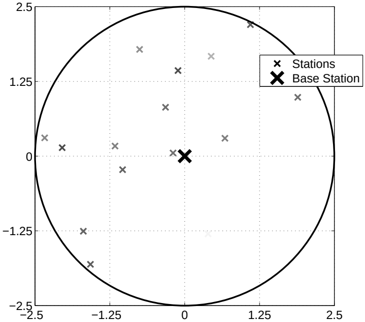
- NSC takes 8.6*ms* to produce a solution, the M^1SC solves the same problem in 26.6*ms* and the M^2SC takes 23.4*ms* to finish.
    - Second–order approximation reduces the computational complexity by more than 10%.
    - The application of the approximations almost triples the computational complexity.
        - Mainly because the approximate algorithms use numerical search.
- Values of C:
    - NSC: $C = 0.735$.
    - M^1SC: $C = 0.734$.
    - M^2SC: $C = 0.735$.
    - M^2SC is more accurate.
        - The M^1SC causes 0.16% error in the aggregate capacity whereas the M^2SC is accurate up to four decimal places.

Experiment I: MSC vs. NSC

- M^1SC vs. NSC:
  - Error in p_i:
    - Mean: 11.50%, Minimum: 0.08%, Maximum: 52.08%.
  - Error in C_i:
    - Mean: 11.70%, Minimum: 0.085%, Maximum: 53.00%
- M^2SC vs. NSC:
  - Error in values of the p_is and the C_is is zero per cent up to **four decimal places**.

Experiment II: M^1SC vs. M^2SC

- While the previous experiment reduced the number of classes to one, in order to be able to also include NSC in the procedure, this experiment considers a truly multiple–class system.
- Darkness of each mobile station demonstrates the corresponding value of α_i (the darker a stations is, the higher the corresponding value of α_i is)

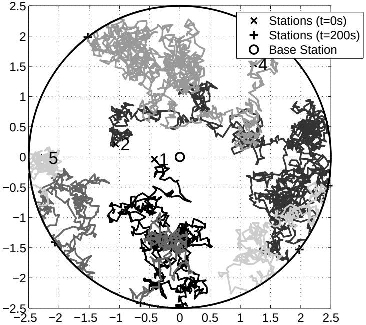


- M^1SC takes about 29.7ms to solve this problem, whereas the M^2SC demands 28.1ms (about 5% less).
- Difference in C: 1.09%.

Experiment III: Dynamic Analysis

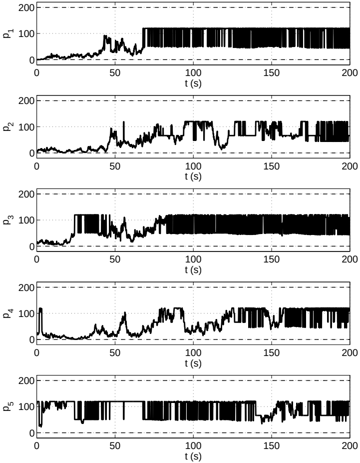
M^2SC was observed to be more accurate and faster.

- The movements of $M = 5$ stations in a cell are simulated and M^2SC is solved at fixed time steps.
- Movements are modeled as a discrete random walk with the speed at each moment chosen based on a uniform random variable between zero and $5Km/h$

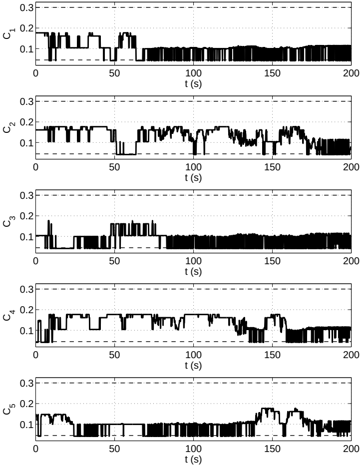


In a time span of $T = 200s$, the resulting problem is solved every $dt = 100ms$.

Experiment III: Dynamic Analysis

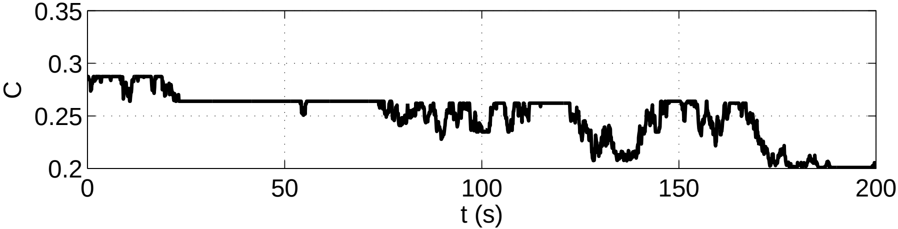


Transmission Powers



Capacities

Experiment III: Dynamic Analysis



Aggregate Capacity during the Experiment

Conclusions

- The maximiziation of the aggregate capacity of the uplink in a single–cell **multiple–class** CDMA system was analyzed.
- Contrary to the previous studies, which assume identical constraints for all the mobile stations, different classes of service were considered.
- It was shown that, through using approximations the new problem can be solved using linear or quadratic programming.
- Second–degree approximation yields a more accurate outcome while it overestimates the capacities and therefore may result in spurious results.
  - No such incidence was observed during the experimental analysis of the proposed method.
- First–order approximation is conservative but induces more error.
- Both algorithms are well inside a 5%–error margin.
- The proposed algorithms are computationally more expensive than their single–class counterparts.
- Experimental analysis of the problem in three different settings was presented.

Thank You for Your Attention,
Any Questions?